

1d plume model based on equations in Joe Kordzi's Basic program. The solution is not coupled, but instead individual components are added together. Individual parts include regional groundwater velocity, operational plume, buoyancy, dispersion, and diffusion.

Facility: Corsicana Technologies
Case: 3 - k=191 md 500years

1. Define Units

$$\begin{aligned} \text{cp} &:= .01 \cdot \text{poise} \\ g &= 9.807 \text{ m} \cdot \text{s}^{-2} \\ \text{md} &:= 7.32441 \cdot 10^{-8} \cdot \frac{\frac{\text{ft}^3 \cdot \text{cp}}{\text{sec}}}{\text{ft}^2 \cdot \frac{\text{psi}}{\text{ft}}} \\ \text{gal} &:= 0.1336894 \text{ ft}^3 \\ \text{acre} &:= 43560 \cdot \text{ft}^2 \end{aligned}$$

2. Reservoir and 10,000 Year Plume Demonstration Parameters

$$\begin{aligned} \rho_{go} &:= 61.97 \cdot \frac{\text{lb}}{\text{ft}^3} & \rho_{gi} &:= 61.61 \cdot \frac{\text{lb}}{\text{ft}^3} \\ k &:= 191 \cdot \text{md} \quad \text{permeability} & \rho_{gi} &= 0.987 \cdot \frac{\text{gm}}{\text{cm}^3} \quad \text{injectate density} \\ \phi &:= 0.20 \quad \text{porosity} & \rho_{go} &= 0.993 \cdot \frac{\text{gm}}{\text{cm}^3} \quad \text{formation fluid density} \\ h &:= 36 \cdot \text{ft} \quad \text{net thickness} & \text{dip} &:= 200 \cdot \frac{\text{ft}}{2.355 \cdot \text{mi}} \quad \text{taken from structure map} \\ \text{cumvolume} &:= 50 \cdot \frac{\text{gal}}{\text{min}} \cdot 30 \cdot \text{yr} & \text{cumvolume} &= 7.889 \times 10^8 \text{ gal} \quad \text{cumulative injection volume} \\ \mu &:= 0.479 \cdot \text{cp} \quad \text{viscosity} & \theta &:= 0.92146586 \text{ deg} \quad \sin(\theta) = 0.016082 \\ \Delta t &:= 500 \cdot \text{yr} \quad \text{plume drift time} & D_0 &:= 4.8 \cdot 10^{-5} \cdot \frac{\text{cm}^2}{\text{s}} \quad \text{free water diffusivity} \\ \alpha_T &:= 16 \cdot \text{ft} \quad \text{transverse dispersivity} & \text{CRF} &:= 1 \cdot 10^{-3} \quad \text{concentration reduction factor} \\ \alpha_L &:= 160 \cdot \text{ft} \quad \text{longitudinal dispersivity} & \tau &:= 1 \quad \text{tortuosity} \\ V_{\text{drift}} &:= 0 \cdot \frac{\text{ft}}{\text{yr}} \quad \text{regional groundwater velocity} \end{aligned}$$

3. Operational Plume Radius and Area

$$\text{operational_plume_radius} := \sqrt{\frac{\text{cumvolume}}{\pi \cdot \phi \cdot h}}$$

$$\text{operational_plume_radius} = 2.123 \times 10^3 \text{ ft}$$

calculated operational plume radius

$$\text{Area_of_plume} := \pi \cdot \text{operational_plume_radius}^2$$

$$\text{Area_of_plume} = 14153336.4 \text{ ft}^2$$

calculated operational plume area

$$\text{Area_of_plume} = 324.916 \text{ acre}$$

4. Movement due to Regional Ground Water Velocity

$$\Delta t = 500 \text{ yr} \quad \text{Vdrift} := 0 \cdot \frac{\text{ft}}{\text{yr}}$$

$$\text{Ground_water_movement_distance} = \text{Vdrift} \cdot \Delta t$$

$$\text{Ground_water_movement_distance} = 0 \text{ ft}$$

calculated movement from regional velocity

5. Movement due to density drift from bouyancy

$$\text{Den1} := 4 \cdot \pi \cdot \sqrt{\alpha T \cdot \alpha L} \cdot k \cdot |\rho_{gi} - \rho_{go}| \cdot g \cdot \sin(\theta) \cdot \Delta t$$

$$\text{Den1} = 1.72 \text{ kg} \cdot \text{s}^{-1} \text{ ft}$$

$$\text{Den2} := \phi^2 \cdot \mu \cdot \text{Area_of_plume}$$

$$\text{Den2} = 88.542 \text{ kg} \cdot \text{s}^{-1} \text{ ft}$$

$$\text{Den3} := 4 \cdot \pi \cdot \frac{\sqrt{\alpha T \cdot \alpha L}}{\text{Area_of_plume} \cdot \phi}$$

$$\text{Den3} = 2.17 \times 10^{-4} \text{ ft}^{-1}$$

$$\text{Density_drift_distance} := \frac{\left[1 + \left(\frac{\text{Den1}}{\text{Den2}} \right)^{0.5} \right] - 1}{\text{Den3}}$$

$$\text{Density_drift_distance} = 44.5 \text{ ft}$$

calculated plume movement from bouyant drift

6. Movement due to dispersion and diffusion

$$\tau := 1 \quad \text{tortuosity}$$

$$D0 = 4.8 \times 10^{-9} \text{ m}^2 \cdot \text{s}^{-1} \quad \text{free water diffusivity}$$

CRF = 1×10^{-3} concentration reduction factor

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L := Density_drift_distance + operational_plume_radius;
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$L = 2167.07 \text{ ft}$ operational plume radius and buoyant plume length

$$\text{standard_deviation} := [2 \cdot [(\alpha L \cdot L) + (D0 \cdot \tau \cdot \Delta t)]]^{0.5}$$

standard_deviation= 833.7ft

Iterative Calculations for error function

$$i := 0..600 \quad X_i := 0.01 + (i \cdot 0.001) \quad X_0 = 0.01$$

$$\text{Error_CRF} := 2 \cdot (\text{CRF}) \qquad \text{Error_CRF} = 0.002$$

$$\text{Error_function}_1 := \text{erfc}(X_i) \quad \text{Error_function}_0 = 0.98871658444415$$

$$\text{Difference}_i := \text{Error_function}_i - \text{Error_CRF} \quad \text{Difference}_0 = 0.987$$

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Converge(X, tol) :=
    j ← 0
    while Differencej > tol
        j ← j + 1
    Xj

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Programming loop to determine argument - solve inverse complementary error function

Define arrays for loop

$\text{Converge}(X, 0.000000000000) = 2.186$

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Z := Converge(X, 0.00000000000000)
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Z = 2.186 The argument of the inverse complementary error function

$\Delta r := \sqrt{2} \cdot Z \cdot \text{standard_deviation}$ from the following equation: $\text{CRF} := \text{erfc}\left(\frac{\Delta r}{\text{standard_deviation} \cdot \sqrt{2}}\right)$

$\Delta r = 2577.4 \text{ ft}$ diffusive and dispersive plume component

7. Summary of Results

operational_plume_radius= 2122.5ft

calculated operational plume radius

Ground water movement distance= 0 ft

calculated movement from regional velocity

Density_drift_distance = 44.5ft

calculated plume movement from bouyant drift

$$\Delta r = 2577.4 \text{ ft}$$

diffusive and dispersive plume movement

$\text{total_plume_distance} := \text{operational_plume_radius} + \text{Ground_water_movement_distance} + \text{Density_drift_distance}$

$\text{total_plume_distance} = 4744.5\text{ft}$

total plume movement from all effects

Note: Ran AERAM model with an improved
dispersion factor for arsenic & got
4748 ft after 500 years